THE GROUP OF HOLOEDRIC TRANSFORMATION INTO ITSELF OF A GIVEN GROUP.

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BY PROFESSOR E. HASTINGS MOORE.

§ 1.

Every group G_n of order n determines a substitution-group Γ^{n-1} of degree $n-1$.

Given an (abstract) group G_n of order n, with the elements $s_1 = \text{identity}, s_2, \ldots s_n$. The group G_n is holoedrically isomorphic with itself if a 1-1 correspondence*

$$(1) \qquad (s_1, s_2, \ldots s_n) \sim (s_{i_1}, s_{i_2}, \ldots s_{i_n})$$

can be set up amongst its elements of such a sort that whenever in G_n we have $s_f s_g = s_h$ we have also $s_{i_f} s_{i_g} = s_{i_h}$. Obviously corresponding elements must have the same period.

Attributing an *order* to the correspondence (1) we have the substitution

$$(2) \qquad \sigma = \begin{pmatrix} s_1, & s_2, \ldots s_n \\ s_{i_1}, & s_{i_2}, \ldots s_{i_n} \end{pmatrix}$$

which holoedrically transforms the group G_n into itself, that is, it transforms the system of product relations

$$(3) \qquad s_f s_g = s_h$$

into itself. *The totality of such substitutions σ constitutes a substitution-group Γ^n of degree n,* which I call *the group of holoedric transformation into itself of the original abstract group G_n of order n.*

If we transform the elements of the group G_n through any particular element of G_n we obtain such an holoedric transformation of G_n into itself. The totality of such transformations constitutes a sub-group of Γ^n.

The group Γ^n on the n elements $s_1, s_2, \ldots s_n$ is certainly intransitive, since elements of the same period in G_n are permuted

* This correspondence may always be established by setting properly chosen *generators* in 1-1 correspondence.

amongst themselves by the substitutions σ of Γ^n. Γ^n is isomorphic with various substitution-groups $\Gamma^{n_i}_{(p_i)}$, where a particular $\Gamma^{n_i}_{(p_i)}$ is a substitution-group on the, say, n_i elements of period p_i. It will on occasion happen that Γ^n, $\Gamma^{n_i}_{(p_i)}$ are *holoedrically* isomorphic, in which case the abstract-group-properties of the Γ^n may perhaps be more conveniently studied as they appear in the $\Gamma^{n_i}_{(p_i)}$. *It will always be desirable to replace the Γ^n by the Γ^{n-1} on the $n-1$ elements $s_2, s_3, \ldots s_n$,* since the identity-element s_1 is invariant.

Intending to discuss the general question somewhat further in a subsequent note, I proceed now to consider an instructive example.

§ 2.

The group Γ^7_{168} of holoedric transformation into itself of the Abelian group G_8 whose elements are the identity and seven commutative elements of period two.

The Abelian or commutative group G_8 has the utmost regularity of internal structure. We define it by the generating elements a, b, c, with the generating relations

$$(4) \qquad a^2 = b^2 = c^2 = 1, \quad ab = ba, \quad ac = ca, \quad bc = cb,$$

where 1 is the identity. The elements may have the following notation:

$$(5) \qquad \begin{matrix} & a & bc = d & \\ 1 & b & ca = e & abc = g \\ & c & ab = f & \end{matrix}$$

Clearly, if $k \neq l$ and $kl = m$, the four elements, $1, k, l, m$ form a four-group. Three such elements k, l, m (whose order is immaterial) form, say, a *triple*. The seven elements $a \ldots g$ arrange themselves into seven triples $A \ldots G$, viz.,

$$(6) \qquad \begin{matrix} A = bcd & D = adg & \\ B = cae & E = beg & G = def \\ C = abf & F = cfg & \end{matrix}$$

This triple system Δ_7 so set up on the elements $a \ldots g$ is invariant under a certain substitution-group $\Gamma^7_{(\Delta_7)}$. This $\Gamma^7_{(\Delta_7)}$ is exactly the Γ^7 of holoedric transformation into itself of the G_8, since the Δ_7 fully defines the product relations of the G_8,

at least when we adjoin the system of equations, $a^2=1, \ldots g^2=1$, which is invariant under every substitution on the $a \ldots g$.

The G_8 and the Δ_7 should be used together in studying the Γ'. The extreme simplicity of the internal structure of the G_8 makes the properties of the Γ' almost immediately evident. A substitution σ of Γ' is determined by the elements a', b', c' which it makes correspond to the *generators* a, b, c. Clearly any one of the seven elements may be chosen for a', any other one for b', and any one except the product $a'b'$ (i.e., except the third element of the triple containing a', b') for c'. The Γ' has then the order $7 \cdot 6 \cdot 4 = 168$. The I^7_{168} is doubly transitive on the seven elements. I shall prove it simple and thereby identify it abstractly with *the only existent* simple * group of order 168.

An enumeration of the substitutions σ of the Γ^7_{168} by period and by type † shows the following distribution:

(7)

	Period	1	2	3	4	7	
	Type	1^7	$1^1 \cdot 2^3$	$1 \cdot 3^2$	$1 \cdot 2 \cdot 4$	7	
	Number	1	+ 21 +	56 +	42 +	48	= 168.

The 21 σ of period 2 are conjugate under the I^7_{168}; likewise the 56 of period 3, and the 42 of period 4. As to those of period 7 consider, say.

(8) $$\tau = (abcfdge);$$

this τ does belong to Γ^7_{168} for the triples

(9) $$abf,\ bcd,\ cfg,\ fde,\ dga,\ geb,\ eac$$

lie on its face cyclically and hence the triple system Δ_7 is invariant under τ. Obviously the triples must lie cyclically on the face of every substitution σ of period 7 of the Γ^7_{168}, not necessarily, however, exactly as in τ; there are two and only two possibilities, both of which occur in the list of powers of τ. In τ^1, τ^2, τ^4 the triples lie as in τ, in, say, the *forward* way; in τ^3, τ^5, τ^6 the triples lie as in

(10) $$\tau^{-1} = \tau^6 = (aegdfcb),$$

in, say, the *backward* ‡ way. Thus the 48 σ of period 7 are half

* HOLDER: Die einfachen Gruppen im ersten und zweiten Hundert der Ordnungszahlen. *Mathematische Annalen*, vol. 40, pp. 55–88, 1892.

† The type of a substitution is the partition-symbol of its degree, arising from its complete expression in the cyclic notation. For instance, in the Γ^7_{168} $\sigma = (abeg)(df)$ has the type $1 \cdot 2 \cdot 4$.

‡ It will be noticed that the separation of the powers is according to the character of the exponent as a quadratic residue or non-residue of 7.

of the forward and half of the backward style. All those of the same style are conjugate under the Γ^7_{168}. For let

$$\tau_1 = (t_{11}t_{12}\,..\,t_{17}) \text{ and } \tau_2 = (\tau_{21}\tau_{22}\,..\,\tau_{27})$$

be two of the same style; the triples lie on the faces of τ_1, τ_2 in the same relative positions; the substitution

$$\lambda = \begin{pmatrix} t_{11}t_{12}\,..\,t_{17} \\ t_{21}t_{22}\,..\,t_{27} \end{pmatrix} \tag{11}$$

transforms the triple system Δ_1 into itself and hence belongs to Γ^7_{168}, and it obviously transforms τ_1 into τ_2. Equally clearly those of different style are *not* conjugate under the Γ^7_{168}. Nevertheless the eight cyclic groups of order 7 are conjugate under the Γ^7_{168}.

Any self-conjugate sub-group Γ^7 of the Γ^7_{168} must then contain besides the identity all or none of the substitutions σ of each type of the list (7). The fact that the diophantine equation

$$1 + 21\alpha + 56\beta + 42\gamma + 48\delta = d, \tag{12}$$

where $\alpha, \beta, \gamma, \delta$ are respectively 0 or 1 and where d is a divisor of 168, has only the two solutions

$$(\alpha, \beta, \gamma, \delta; d) = (1, 0, 0, 0; 1), (1, 1, 1, 1; 168) \tag{13}$$

shows that the Γ^7_{168} is simple.* This identifies the group Γ^7_{168}; of which later.

The triple system Δ_1 is given in (5) (6) from the standpoint of the seven elements $a\,..\,g$. I give it again from the standpoint of the seven triples $A\,..\,G$:

$$\begin{matrix} & A & D & \\ & B & E & G \\ & C & F & \end{matrix} \tag{14}$$

$$\begin{matrix} a = BCD & d = ADG & \\ b = CAE & e = BEG & g = DEF \\ c = ABF & f = CFG & \end{matrix} \tag{15}$$

* This method of proving the simplicity of a simple group when its elements and cyclic sub-groups have been classified as to conjugacy under the group was used by KLEIN for the icosahedron G_{60} (Vorlesungen über das Ikosaeder, p. 18, 1884) and has, as is well known, wide application. Is an instance known in which it does not apply?

Obviously (14) (15) exhibit in the $A \ldots G$ a triple system $\Delta_1(A \ldots G)$ exactly * like the triple system $\Delta_1(a \ldots g)$ exhibited by (5) (6).

Now a substitution σ of Γ_{168}^7 permutes the $a \ldots g$, leaving $\Delta_1(a \ldots g)$ invariant; through the $a \ldots g$ it permutes the $A \ldots G$, but nevertheless leaves the $\Delta_1(A \ldots G)$ invariant. Call $\bar{\sigma}$ the substitution on the $A \ldots G$ which permutes the $A \ldots G$ directly, just as σ does indirectly. The totality of these substitutions $\bar{\sigma}$ constitutes a group, the group $\bar{\Gamma}_{168}^7$ of the $\Delta_1(A \ldots G)$, which is clearly on the one hand abstractly identical with the Γ_{168}^7 of the $\Delta_1(a \ldots g)$ and on the other hand holoedrically isomorphic with the Γ_{168}^7 by the correspondence

$$\sigma \sim \bar{\sigma}. \tag{16}$$

This holoedric isomorphism of the abstract Γ_{168}^7 with itself is not that arising from a transformation of the Γ_{168}^7 through one of its own elements. For such a transformation would change the substitution τ (8) of period 7 into another one of the forward style, while (16) gives the correspondence

$$\tau = (abcfdge) \sim \bar{\tau} = (AFGDEBC), \tag{17}$$

where $\bar{\tau}$ is of the backward style.

The simple group Γ_{168} (abstractly, there is but one) first appeared as the substitution-group Γ_{168}^8 connected with the modular equation for the transformation of elliptic functions of order 7. Galois discovered the existence of a resolvent of order 7. As to the development of the theory one may consult the introduction to a memoir of Mr. Gordan.†

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* A triple system Δ_n in n elements can exist only if n has the form $6m+1$ or $6m+3$, say the form t. All triple systems Δ_7 are of the same class; likewise for the Δ_7, Δ_9. Triple systems Δ_t for every t do exist, and indeed for every $t \geq 13$ they fall into at least two distinct classes. As to these theorems these references:

NETTO: Zur Theorie der Tripelsysteme. *Mathematische Annalen*, vol. 42, pp. 143–152, 1893.

MOORE: Concerning Triple Systems. *Mathematische Annalen*, vol. 43, pp. 271–285, 1893.

JAN DE VRIES: Zur Theorie der Tripelsysteme. *Rendiconti del Circolo Matematico di Palermo*, vol. 8, pp. 222–226, 1894.

† GORDAN: Ueber Gleichungen siebenten Grades mit einer Gruppe von 168 Substitutionen. *Mathematische Annalen*, vol. 20, pp. 515–530, 1882.

The triple system as determining invariant of the group Γ^7_{168} was introduced by Mr. Noether.*

The sub-groups of the group † $\Gamma^{\frac{q+1}{q(q^2-1)}}_{2}$ connected ‡ with the modular equation for the transformation of prime order q have been completely enumerated by Mr. Gierster, § thus illuminating Galois' theorem that the modular equation has resolvents of degree q for $q = 5, 7, 11$, but for no $q > 11$, and Kronecker's theorem ‖ that the group $\Gamma^{\frac{q+1}{q(q^2-1)}}_{2}$ is holoedrically isomorphic with itself in two distinct ways, and only ¶ in those ways.

The idea ** of the group Γ^{n-1} of holoedric transformation into itself of a given group G_n is, so far as I know, new. In the preceding sketch of the Γ^7_{168} the perspicuous determination from the Abelian G_8 of the triple system Δ_7 and of the Γ^7_{168} is the only element of novelty.

THE UNIVERSITY OF CHICAGO, *November* 5, 1894.

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* NOETHER: Uber die Gleichungen achten Grades und ihr Auftreten in der Theorie der Curven vierter Ordnung. *Mathematische Annalen*, vol. 15, pp. 89–110, 1879).

† The $\Gamma^{\frac{q+1}{q(q^2-1)}}_{2}$ for $q > 3$ is simple.

‡ *Connected with* is *of*, if the numerical irrationality $\sqrt{(-1)^{\frac{q-1}{2}} \cdot q}$ is adjoined

§ GIERSTER: Die Untergruppen der Galois'schen Gruppe der Modulargleichungen für den Fall eines primzahligen Transformationsgrades. *Mathematische Annalen*, vol. 18, pp. 319–365, 1881.

‖ KRONECKER: *Monatsberichte der Berliner Akademie*, 1861.

¶ As GIERSTER adds, *loc. cit.* p. 356.

** This concept and an application of it to the Γ^7_{168} were communicated to the Mathematical Club of the University of Chicago in a paper presented January 19, 1893, entitled "An existence-proof of the simple group of order 168 as a group of substitutions on 7 letters."

ON THE NON-PRIMITIVE SUBSTITUTION-GROUPS OF DEGREE TEN.

BY DR. G. A. MILLER.

A LIST of the non-primitive substitution-groups of degree ten is given in the *Quarterly Journal of Mathematics*, vol. 27, pp. 40–42. By comparing with this list the theorems given below it will be found that the following six groups should be deleted:

$$\begin{aligned}
200_3 &= \{(abcde)_{20}(fghij)_{20}\}\text{quad}_1(afbg\,.\,chej\,.\,di)\\
200_4 &= (abcde)_{10}(fghij)_{10}(afbg\,.\,chej\,.\,di)\\
200_5 &= (abcde)_{10}(fghij)_{10}(ajbh\,.\,cfeg\,.\,di)\\
100_2 &= \{(abcde)_{10}(fghij)_{10}\}\text{dim}(ajbh\,.\,cfeg\,.\,di)\\
50_2 &= (abcde)_5(fghij)_5(af\,.\,bh\,.\,cj\,.\,dg\,.\,ei)\\
50_3 &= (abcde)_5(fghij)_5(af\,.\,bj\,.\,ci\,.\,dh\,.\,eg)
\end{aligned}$$

Theorem I. There are five and only five non-primitive groups of degree ten and order two hundred.

The only bases that can be used are

$$\begin{aligned}
100_4 &= \{(abcde)_{20}(fghij)_{20}\}\text{quad}_1\\
100_5 &= \{(abcde)_{20}(fghij)_{20}\}\text{quad}_2\\
100_3 &= (abcde)_{10}(fghij)_{10}
\end{aligned}$$

Since all of these involve one and only one cycle of five letters and its powers in each system and $(abcde)_{20}(fghij)_{20}$ contains all the substitutions, whose degree does not exceed ten, which transform

$$(abcde)\ \text{cyc}\ (fghij)\ \text{cyc}$$

into itself, it follows that there cannot be more than four hundred substitutions which transform any one of these bases into itself and also interchange the systems. Two hundred of these are positive and two hundred negative.

The substitution

$$af\,.\,bg\,.\,ch\,.\,di\,.\,ej$$

may evidently be used to generate a tail to each one of these bases,* and as these bases together contain two hundred dif-

* The conditions which such a substitution must satisfy to generate a tail containing as many substitutions as the head contains, in case the letters are divided into two equal systems, are given in the article to which we referred in the first paragraph. They are: (1) it must transform the base into itself; (2) it must interchange the systems of intransitivity; and (3) its square must be found in the base.

ferent substitutions we must employ all the possible negative substitutions in making up the three tails for the given bases, or heads. As the groups just obtained are distinct, we have now three non-primitive groups of degree ten and order two hundred.

It should be observed that since all the four hundred possible substitutions transform $(abcde)$ cyc $(fghij)$ cyc into itself, we may prove that one of these substitutions transforms any one of the given bases into itself by showing that it transforms the remaining generating substitutions into substitutions of the base. In the first and second cases we may select one substitution as the remaining generating substitution, but in the third case we cannot select less than two.

We now inquire whether the one hundred negative substitutions which are not employed in each individual case can be used as a tail in that case, and, if so, whether the group thus obtained is distinct from those already given. In the first and third cases one of these remaining negative substitutions is

$$af \cdot bg \cdot ch \cdot di \cdot ej \times aceb \cdot fgjh^* = ag \cdot bj \cdot cf \cdot di \cdot eh$$
$$= fgjh^{-1} \times af \cdot bg \cdot ch \cdot di \cdot ej \times fgjh$$

Since $fgjh$ transforms these bases into themselves, it follows that

$$ag \cdot bj \cdot cf \cdot di \cdot eh$$

may be used to generate a tail to each of these heads, but that the groups thus obtained are *not* distinct from the three already given.

In the second case one of these one hundred remaining negative substitutions is

$$af \cdot bg \cdot ch \cdot di \cdot ej \times abcd \cdot fghi = agci \cdot bhdf \cdot ej$$

Since $agci \cdot bhdf \cdot ej$ satisfies the three necessary and sufficient conditions specified in the foot-note, it may be used to generate a tail to the second head, and it remains only to show that the group thus generated is distinct from the three already found. To do this it is only necessary to show that it is distinct from the one of these three which has the same base. If it were not distinct from this it would contain a substitution of the form

$$af \cdot bg \cdot ch \cdot di \cdot ej$$

* We have employed the notation of Professor CAYLEY as given in the *Quarterly Journal of Mathematics*, vol. 25, p. 77, in writing $(abcde)_{25}$ and $(abcde)_{10}$. To obtain the notation employed in the list to which we referred we have to transform with respect to $cd \cdot hi$.

and beginning with $af.$; for if a particular substitution of this form would not begin with $af.$ it could be made to do so by transforming with respect to some substitution in $(fghij)$ cyc. The only substitutions in our group beginning with $af.$ are

$$af.bieh.cgdj$$
$$af.bjeg.cidh$$
$$af.bgej.chdi$$
$$af.bhei.cjdg$$

and as these do not have the required form our group must be distinct from the three which have already been given.

We now proceed to the positive substitutions. One of these is

$$afbgchdi.ej$$

Its square is found only in the first base, and it is easily seen that it may be used to generate a tail to this base, thus giving rise to the fifth and last group included in our theorem. For one of the remaining one hundred positive substitutions is

$$afbgchdi.ej \times abcd.fgjh = agdibjeh.cf$$
$$= \overline{bdec.fgihj}^{-1} \times afbgchdi.ej \times bdec.fgihj$$

Since $bdec.fgihj$ transforms the first base into itself, it follows that a group is obtained by combining $agdibjeh.cf$ with this base, which is not distinct from the preceding group. The two hundred positive substitutions form two conjugate tails to the first base, hence all their squares are found in this base. From this it readily follows that all these positive substitutions are of the form $afbgchdi.ej$, and that none of their squares are found in either of the other bases. This completes the proof of the theorem.

Theorem II. There are two and only two non-primitive groups of order one hundred and degree ten.

The only possible base is

$$\{(abcde)_{10}(fghij)_{10}\}\text{dim.}$$

From what precedes it is readily seen that the only admissible substitutions for tails are the two hundred negative substitutions which have already been employed.

$$af.bg.ch.di.ej$$

can be used to generate a tail, and we thus employ fifty of the possible substitutions. By transforming this group with respect to *fghi* we obtain a group having the same head but a different tail, as

$$ag \,.\, bh \,.\, ci \,.\, df \,.\, ej$$

is not contained in the preceding group. We have now used one hundred of the possible substitutions and obtained only one distinct group. The substitution

$$agci \,.\, bhdf \,.\, ej$$

which we used in a preceding case is not included in these one hundred, and it may be used to generate a tail, which may be easily proved by the method employed under the first theorem to be distinct from the tails already found. We thus obtain a second distinct group, and if we transform this with respect to

$$fghi$$

we obtain a group which has the same head but a different tail from the last, since

$$ahcf \,.\, bidg \,.\, ej$$

is not found in the three preceding tails. We have now exhausted the two hundred possible substitutions and found only two distinct groups.

Theorem III. There is one and only one non-primitive group of degree ten and order fifty.

The only base which can be used is

$$(abcde)\ \text{cyc}\ (fghij)\ \text{cyc}.$$

Since it has been proved that all the positive substitutions which interchange these cycles are of the form

$$afbgchdi \,.\, ej,$$

none of their squares is contained in this base and they can therefore not be used to generate a tail. Under Theorem I. we found the following negative substitutions:

$$af \,.\, bieh \,.\, cgdj$$
$$af \,.\, bjeg \,.\, cidh$$
$$af \,.\, bgej \,.\, chdi$$
$$af \,.\, bhei \,.\, cjdg$$

By transforming these with respect to

$$(fghij) \text{ cyc}$$

we find that there are twenty different substitutions of this form and beginning with

$$a\beta. \quad \beta = f, g, h, i, j$$

among the two hundred possible negative substitutions. By transforming these twenty with respect to

$$(abcde) \text{ cyc}$$

we find one hundred such negative substitutions, which are all different and begin with

$$\alpha\beta. \quad \begin{matrix} \alpha = a, b, c, d, e \\ \beta = f, g, h, i, j \end{matrix}.$$

Since the square of none of these is found in our base, there cannot be more than one hundred substitutions available to construct tails to it.

$$af \,.\, bg \,.\, ch \,.\, di \,.\, ej$$

may be used to generate one tail, and we proceed to show that the twenty-five substitutions obtained in this way may be transformed into the seventy-five other possible substitutions by means of substitutions which transform the base into itself.

If we transform the given group with respect to

$$gj \,.\, hi$$

we obtain a new group, as $af \,.\, bj \,.\, ci \,.\, dh \,.\, eg$ is not found in the preceding group, there being only one substitution in it beginning with af.: then we may transform both of these groups with respect to

$$fghi$$

and obtain two new groups, as $ag \,.\, bh \,.\, ci \,.\, df \,.\, ej$ is not found in either of the preceding groups, since the only substitutions in these groups beginning with ag. are

$$ag \,.\, bi \,.\, cj \,.\, dh \,.\, ef$$
$$ag \,.\, bf \,.\, ch \,.\, dj \,.\, ei$$

and $ag \,.\, bj \,.\, cf \,.\, di \,.\, eh$ is not found in any one of the three preceding groups, since all the substitutions in these groups which begin with ag . are

$$ag \,.\, bi \,.\, cj \,.\, dh \,.\, ef$$
$$ag \,.\, bf \,.\, ch \,.\, dj \,.\, ei$$
$$ag \,.\, bh \,.\, ci \,.\, df \,.\, ej$$

Since $gj \,.\, hi$ and $fghi$ are contained in $(abcde)_{20}(fghij)_{20}$ they transform the given base into itself, we have therefore exhausted the possible one hundred substitutions and found only one distinct group. Our theorem is thus proved.

An independent proof of these theorems may be outlined as follows. The non-primitive groups in question must contain $(abcde)$ cyc $(fghij)$ cyc, as a common self-conjugate subgroup. They must therefore be subgroups of a group whose order is $20 \cdot 20 \cdot 2 = 800$ and this group must have a 25-1 correspondence to some non-primitive group in eight letters, since a different letter corresponds to each substitution of the form $abcde$. The required group is easily found to be

$$(ae \,.\, bf \,.\, cg \,.\, dh)(abcd) \text{ cyc } (efgh) \text{ cyc}$$

Since this group has the following non-primitive subgroups

Order.	Number of subgroups.
32	1
16	2
8	5
4	2
2	1

—which are not conjugate with respect to substitutions in it, there must be the same numbers of distinct non-primitive subgroups of the required type in the given group of order 800, and the orders of these are found by multiplying the orders of the given subgroups by 25. The numbers of the subgroups of the last three orders constitute the proofs of our theorems.

It may be remarked that the list to which we referred in the first paragraph contains all the groups which we have just found, but that it contains, in addition to these, six which are not distinct from others in the list. 200_5 and 200_6 are identical, but in all other cases the groups which should be deleted are conjugate to others already in the list.

MICHIGAN UNIVERSITY, *October*, 1894.

BRIEFER NOTICES.

HEINRICH HERTZ: *Gesammelte Werke.* Band III: *Die Prinzipien der Mechanik in neuem Zusammenhange dargestellt.* Mit einem Vorworte von H. von HELMHOLTZ. Leipzig, Barth, 1894. 8vo. xxx + 312 pp.

Only the third volume of this edition of the collected works of the late Heinrich Hertz has so far appeared; but the two other volumes were promised for the fall of 1894 and will probably soon be ready.

The first volume will contain the papers written during the years 1879–1884, including the doctor dissertation and the address on light and electricity delivered at the congress of German naturalists at Heidelberg, and also some papers of very recent date. Ph. Lenard, of Bonn, is the editor. Volume II will be a reprint of the papers previously published under the title "Untersuchungen über die Ausbreitung der elektrischen Kraft" (Leipzig, Barth, 1892), of which an English translation was recently issued ("Electric Waves: being researches on the propagation of electric action with finite velocity through space;" authorized English translation by D. E. Jones, with a preface by Lord Kelvin. London, Macmillan, 1894).

The third volume is also edited by Professor Lenard. It contains an exceedingly interesting and original treatise, or rather memoir, on the principles of theoretical mechanics and mathematical physics which was composed during the last three years of the author's life. A. Z.

HERMANN GRASSMANN: *Gesammelte mathematische und physikalische Werke.* Auf Veranlassung der mathematisch-physischen Klasse der kgl. sächsischen Akademie der Wissenschaften und unter Mitwirkung der Herren: Jakob Lüroth, Eduard Study, Justus Grassmann, Hermann Grassmann der Jüngere, Georg Scheffers, herausgegeben von Friedrich Engel.—Ersten Bandes erster Theil: *Die Ausdehnungslehre von 1844 und die Geometrische Analyse.* Unter Mitwirkung von Eduard Study herausgegeben von Friedrich Engel. Mit einem Bilde Grassmanns. Leipzig, Teubner, 1894. 8vo. xvi + 436 pp.

The idea of a new and complete edition of Hermann Grassmann's mathematical works was first suggested by Professor Felix Klein, of Göttingen. After obtaining the consent of Grassmann's surviving relatives he personally laid the matter before the Saxon Academy of Sciences (in October, 1892), insisting in particular on the desirability of including in the proposed edition all hitherto unpublished papers of any importance. Upon his recommendation the Academy elected a

committee to devise ways and means, and entrusted Professor Engel, of Leipzig, with the chief editorship of the undertaking.

As appears from the title given above and from the preface, Professor Engel has succeeded in securing for his task the active co-operation of some of the ablest mathematicians of Germany. In the volume before us, which constitutes the first half of the first of the three volumes contemplated, the editorial work seems to be done throughout with the utmost care and with an almost philological minuteness. If the same standard be maintained in the remaining portions,—and there is no reason for doubting this,—the present edition, which appears just fifty years after the first Ausdehnungslehre, will remain for all time the *editio princeps*.

It was certainly wise to take the liberty of making slight changes here and there in the phraseology of the original and of improving the subdivision into paragraphs. All important changes are clearly indicated; and even the pagination of the original editions is marked in the margin. Over twenty closely-printed pages of notes of an explanatory, critical, and historical nature are added in the present volume by the editors (Engel and Study). A portrait of the author adorns the volume.

The second half of the first volume, containing the Ausdehnungslehre of 1862, is promised to be issued early in 1895. The author's son, Hermann Grassmann, Jr., of Halle, will assist Professor Engel in editing this part.

The second volume will contain Grassmann's printed papers, together with some hitherto unpublished articles. Dr. Scheffers, of Leipzig, and Professor Study, of Bonn, will assist in editing the geometrical papers. The articles on mechanics, both published and unpublished, have already been prepared for publication by Professor Lüroth, of Freiburg, author of a valuable "Grundriss der Mechanik" (München, Ackermann, 1881), based on Grassmann's ideas.

The thesis on the theory of the tides, written in 1840 and referred to in the preface to the first Ausdehnungslehre (p. v), but never published, will appear in the third volume, together with the remaining posthumous papers. Justus Grassmann, of Brandenburg, will edit the memoir on the tides. Professor Engel promises to contribute a biography of Grassmann and a brief connected account and criticism of his scientific work. Finally, Professors Schlegel, of Hagen, and Mehmke, of Darmstadt, are preparing for this last volume a complete bibliography of works having reference to Grassmann's ideas.

It should be noticed that the present edition is not the work of one-sided partisans of Grassmann's methods. Professors Engel and Study, in particular, have on former occasions, while expressing their appreciation of the value of

Grassmann's work, pointed out its necessary limitations (see, for instance, Lie and Engel's "Theorie der Transformationsgruppen," vol. III, Leipzig, Teubner, 1893, p. 534 and p. 748, Study's "Methoden zur Theorie der ternären Formen," Leipzig, Teubner, 1889, p. 202, and Study's "Massbestimmung extensiver Grösen, Wien, 1885). This critical spirit pervades the notes to the present volume and makes them really helpful to the student (see, in particular, the note on pp. 404–406). It is the modern theory of transformation-groups which furnishes the final criterion for a judicious estimate of the Ausdehnungslehre. A. Z.

Jahresbericht der Deutschen Mathematiker-Vereinigung. Vol. III (1893). Berlin, Reimer.

While this volume does not seem to have appeared as yet, a brief extract has been sent out which contains the general report on the Munich meeting, the financial report up to the spring of the present year, and the list of members. Besides an obituary of the late E. E. Kummer, by E. LAMPE, the third volume will contain two longer reports, viz.: "On the development of the theory of algebraic functions in the past and up to the present time," by BRILL and NÖTHER, and "On the development and main problems of the theory of girder-frames (einfaches Fachwerk)," by HENNEBERG. Abstracts will also be given of the following papers read at the meeting:

DYCK (Munich): Introductory address on the mathematical exhibition.

HAAS (Vienna): On an apparatus exhibiting precession.

HILBERT (Königsberg): Two new proofs of the possibility of resolving the numbers of a "Körper" into "Primideale."

MEHMKE (Darmstadt): On calculating-machines and similar appliances.

BJERKNES (Christiania): Analogies between physical (in particular electrical and magnetic) and purely mechanical phenomena.

JUKOVSKY (Moscow): Geometrical interpretation of the case discussed by Hess in the motion of a heavy rigid body about a fixed point.

H. WIENER (Halle): Further remarks on the foundations and the systematic development of geometry.

M. SIMON (Strassburg): Proof of the proposition that a Lobachevsky space contains a double infinity of spheres of infinite radius.

PRINGSHEIM (Munich): On Taylor's series.

BRUNN (Munich): Contribution to the *analysis situs*.

BAUSCHINGER (Munich): On the testing of the strength of materials.

FREYBERG (Dresden): Töpler's apparatus for illustrating the statics and dynamics of rigid bodies.

VON LOMMEL (Munich): On lines of magnetic force.

SCHAPIRA (Heidelberg): Iteration as the fundamental idea in mathematical operations.

FRICKE (Göttingen): On the moduli of a Riemann surface.

DOEHLEMANN (Munich): On the theory of the null-system.

BURKHARDT (Göttingen): On the theory of Cremona transformations.

SCHAPIRA (Heidelberg): On symmetric and alternating quadratic forms. A. Z.

Proceedings of the American Association for the Advancement of Science for the forty-second meeting, held at Madison, Wis., August, 1893. Salem, 1894. 8vo. cxvi + 384 pp.

Besides Professor C. L. Doolittle's vice-presidential address to Section A on "Variations of latitude," this volume contains brief abstracts of a paper by Professor F. H. Loud, "A construction for the imaginary points and branches of algebraic loci," of a paper by Professor E. W. Hyde, "The screw as a unit in a Grassmannian system of the sixth order" (published in full in the *Annals of Mathematics*, vol. 8), and of a paper by Professor Mansfield Merriman, "On the possibility of the algebraic solution of the general equation of the fifth degree." Several other papers read before Section A are given only by title. A. Z.

Le Livret de l'Étudiant de Paris. Publié sous les auspices du Conseil générale des Facultés. 1894–1895. Paris, Delalain Frères, 1894. 12mo. 158 pp.

A small volume bearing the above title has just been published by the General Council of the Faculties of Paris on behalf of those institutions which taken together are regarded as constituting the *Académie de Paris.* In the preface we read: "The General Council of the Faculties deems it useful to bring together the various information necessary for each student. . . . In giving to the students of Paris greater facilities for informing themselves as to both the rules to which they are subject and the means of study which are at their disposal, in showing them all the resources which the faculties and schools offer for higher intellectual culture, the publishers hope to contribute, on their part, to the rapidly-progressing development of the *University of Paris.*" The matters presented are: (1) programmes of the several faculties and the school of pharmacy; (2) announcements of other institutions of higher instruction; (3) academic regulations;

(4) addresses of professors; (5) students' societies; (6) index of courses; (7) index of regulations; (8) index of names. Among the courses announced by the Faculty of Science for the semester beginning November 12, 1894, are: Darboux, Theory of rectilinear congruences and of the deformation of surfaces,—Picard, Study of curves defined by differential equations. Investigation of linear differential equations, particularly from the point of view of the theory of continuous and discontinuous groups,—Appell, General laws of equilibrium and motion,—Tisserand, Theory of the motions of the satellites of Jupiter and Saturn,—Poincaré, Study of the theory of the Newtonian potential,—Boussinesq, Study of the equilibrium and motion of elastic solids and of the resistance, either statical or dynamical, of these bodies. Supplementary courses, including conferences upon subjects required for the *Agrégation des Sciences mathématiques*, are given by Messrs. Raffy, Koenigs, Andoyer, Puiseux, Painlevé, and others. T. S. F.

NOTES.

A REGULAR meeting of the AMERICAN MATHEMATICAL SOCIETY was held Saturday afternoon, November 24, at three o'clock, the President, Dr. McClintock, in the chair. Professor John H. Kleinheksel, of Hope College, Holland, Mich., and Mr. Henry Freeman Stecker, of the University of Wisconsin, Madison, Wis., having been duly nominated, and being recommended by the Council, were elected to membership. The following papers were presented:

(1) Mr. C. S. PEIRCE: "Rough notes on geometry. Constitution of real space."

(2) Professor E. HASTINGS MOORE: "The group of holoedric transformation into itself of a given group."

In the absence of Professor Moore, his paper was read by the Secretary. It appears in the present number of the BULLETIN, page 61. Mr. Vatslov A. Hlasko, who was present by invitation, exhibited and explained a model, which he had constructed, representing a three-dimensional projection of the regular figure of four dimensions bounded by six hundred tetrahedra.

WE record with regret the death, on November 8 last, of William Curns Lawrence Gorton, professor of mathematics and astronomy at the Woman's College of Baltimore.

DR. H. T. EDDY has accepted an appointment as professor

of mechanics and engineering at the University of Minnesota, Minneapolis.

THE annual meeting of the London Mathematical Society was held on the evening of November 8. The following officers were elected for the coming year: President, Major Macmahon; Vice-Presidents, Prof. M. J. M. Hill, Mr. A. B. Kempe, Mr. A. E. Love; Treasurer, Dr. J. Larmor; Secretaries, Mr. M. Jenkins, Mr. R. Tucker; other members of the Council, Mr. A. B. Bassett, Mr. G. H. Bryan, Lieut.-Col. J. R. Campbell, Lieut.-Col. A. J. C. Cunningham, Prof. E. B. Elliott, Dr. J. W. L. Glaisher, Prof. A. G. Greenhill, Dr. E. W. Hobson, Prof. W. H. H. Hudson. An address entitled "Mathematics" was delivered by Mr. A. B. Kempe, the retiring president.

T. S. F.

THE meeting of the German Mathematical Association was held this year at Vienna, during the last week of September. The following is a partial list of the papers read:

CZUBER (Vienna): A symbolic calculus on curves of deficiency one.

DYCK (Munich): On the theory of characteristics of systems of functions.

GEGENBAUER (Vienna): On prime numbers.

GORDAN (Erlangen): The resolution of curves into straight lines.

KLEIN (Göttingen): On homogeneous linear differential equations of the second order with two variables.

KOHN (Vienna): Generalization of a fundamental concept of the geometry of position.

KÖNIGSBERGER (Heidelberg): On the theory of partial differential equations.

MANDL (Prossnitz): A criterion for the irreducibility of integral functions.

MAYER (Clausthal): Formation of resultants in trigonometry.

NAGY (Rome): (*a*) On the Jevons-Clifford problem; (*b*) Introduction to the theory of logical forms.

PESCHKA (Vienna): On the desirability of lectures on descriptive geometry in the universities.

SCHAPIRA (Heidelberg): (*a*) On homoidal functions; (*b*) On invariants of transformations and iterations.

SCHLESINGER (Berlin): On Poincaré's representation of the integrals of a homogeneous linear differential equation of the second order as single-valued functions of one parameter.

SIMONY (Vienna): (*a*) On the classification of knots; (*b*) On the relation of topology to geometry and mechanics.

STÄCKEL (Halle): Application of Lie's theory of groups to dynamics.

STUDNIČKA (Prague): On functions of a quaternion variable.

TAUBER (Vienna): Poisson's integral and its conjugate.

VASILIEV (Kazàn): Lobachevsky's views on the theory of parallels before 1826.

WIRTINGER (Vienna): On the connection of Kummer's surface with the projective generation of plane quartics having a double point.

ZINDLER (Vienna): A new method for generating a linear complex through two rotations.

ZSIGMONDY (Vienna): On congruencies having no roots with respect to a prime modulus.

THE magnificent library of the late Prince Baldassare Boncompagni, of Rome, consisting of 18,000 printed works and 600 manuscripts, probably the most valuable private collection on the history of mathematics, will be offered for sale. It had been the Prince's intention to bequeath the whole collection to the library of the Vatican. But it appears that he died before he could accomplish his purpose.

THE more important mathematical works announced as in press by Messrs. Gauthier-Villars et Fils are in great part continuations. The second volume of J. TANNERY and J. MOLK'S "Éléments de la théorie des fonctions elliptiques" concludes the differential calculus of these functions. The third volume of FERMAT'S works will contain, besides supplements to Vol. II, French translations of Fermat's Latin papers, of his correspondence with Wallis, and of Jacques de Billy's "Inventum novum." The second part of MERAY'S "Leçons nouvelles sur l'analyse infinitésimale et ses applications géométriques" is devoted to a monographic study of the principal functions of a single variable. The second volumes are also in press of P. APPELL'S "Traité de mécanique rationnelle," of B. BAILLAUD'S Cours d'astronomie," and of B. NIEWENGLOWSKI'S "Cours de géométrie analytique." The fourth and last volume of F. TISSERAND'S "Traité de mécanique céleste" will treat the perturbations of the minor planets by the methods of Cauchy, Hansen, and Gyldén, and will develop the theory of the motion of the satellites. A. Z.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

BÔCHER (M.). Ueber die Reihenentwickelungen der Potentialtheorie. Mit einem Vorwort von Felix Klein. Leipzig, Teubner, 1894. 8vo. viii and 258 pp. Illustrated. Mk. 8.00

BÖKLE (C.). Ueber den Zusammenhang der Dupin'schen Cykliden mit den confocalen Flächen zweiter Ordnung. Mit einem Anhang: Ueber Kreis- und Kugelmultipel. [Progr.] Frankenthal, 1893. 8vo. 32 pp.

DARBOUX (G.). Leçons sur la théorie générale des surfaces et les applications géométriques du calcul infinitésimal. (En 4 volumes.) Vol. III: Lignes géodésiques et courbures géodésiques; paramètres différentiels; déformation des surfaces. Paris, Gauthier-Villars, 1894. 8vo. viii and 512 pp. Fr. 15.00

FOERSTER (W.). Rede über das Zusammenwirken von Bessel, Encke und Alexander von Humboldt unter der Regierung Friedrich Wilhelm III. [Progr.] Berlin, 1894. 4to. 21 pp.

HECKHOFF (M.). Die Schraubenflächen. Eine monographische Studie. [Diss.] Tübingen, 1894. 8vo. 43 pp.

HENKE (R.). Ueber die Methode der kleinsten Quadrate. 2te, unveränderte Auflage. Nebst Zusätzen. Leipzig, Teubner, 1894. 8vo. 5 and 77 pp. Mk. 2.00

KÖLMEL (F.). Ableitung der verschiedenen Formen der Kurven dritter Ordnung durch Projektion und Klassifikation derselben. I. [Progr.] Ettenheim, 1894. 4to. 12 pp. Illustrated.

LÁSKA (W.). Sammlung von Formeln der reinen und angewandten Mathematik. Lieferung III. 2te Hälfte. Braunschweig, Vieweg, 1894. 8vo. Mk. 7.50

MATHEMATICAL QUESTIONS AND SOLUTIONS; from "Educational Times," etc., edited by W. J. C. Miller. Vol. 61. London, Hodgson, 1894. 8vo. Boards. 6s. 6d.

MÜLLER (K. E.). Ueber die algebraischen Integralfunktionen von Systemen algebraischer Differentialgleichungen. Die Reduktion der algebraischen Integralfunktionen auf die Form der rechten Seiten ihrer normalen algebraischen Differentialgleichungssysteme. [Progr.] Lahr, 1894. 4to. 15 pp.

OBENBRAUCH (F. J.). Monge, der Begründer der darstellenden Geometrie als Wissenschaft. Eine mathematisch-historische Studie. Theil II. Brünn, 1894. 8vo. 20 pp. Mk. 1.00

Oss (S. L. von). Die Bewegungsgruppen der regelmässigen Gebilde von vier Dimensionen. [Diss.] Giessen, 1894. 8vo. 38 pp.

Pasch (M.). Ueber den Bildungswerth der Mathematik. [Progr.] Giessen, 1894. 4to. 18 pp.

Pokorny (A.). Construction der gemeinschaftlichen Punkte und Tangenten zweier Kegelschnitte für den Fall, dass zwei solche Punkte oder Tangenten gegeben sind. [Progr.] Troppau, 1894. 8vo. 9 pp.

Pruvost (E.). Leçons de géométrie analytique à l'usage des élèves de la classe de mathématiques spéciales et des candidats à l'École normale supérieure et à l'École polytechnique. (En 2 volumes.) Vol. I: Géométrie plane. Vol. II: Géométrie dans l'espace. Paris, Dupont, 1894. 8vo. Fr. 21.50

Rebière (A.). Les femmes dans la scienc . Paris, Nony, 1894. 8vo. Fr. 1.50

Schumacher (J.). Ueber Determinanten. (Festschrift.) Neustadt, 1894. 8vo. 21 pp.

Schwarz (H. A.). Formules et propositions pour l'emploi des fonctions elliptiques, d'après les leçons èt des notes manuscrites de K. Weierstrass. Traduit de l'allemand par H. Padé. 1re partie. Paris, Gauthier-Villars, 1894. 4to. Fr. 12.00

Spindeler (K.). Ein Beitrag zur Einführung in das Gebiet der räumlichen Configurationen. [Progr.] Diedenhofen, 1894. 4to. 31 pp.

Uzielli (G.). La vita e i tempi di Paolo dal Pozzo Toscanelli; ricerche e studi. Con un capitulo sui lavori astronomici di Toscanelli da G. Celoria. Firenze, 1894. 745 pp. 11 plates. Mk. 62.00

Vasiliev (A.). Nicolái Ivánovich Lobachévsky. Address pronounced at the commemorative meeting of the Imperial University of Kasán, October 22, 1893. Translated by G. B. Halsted. Austin, 1894. 8vo. 40 pp.

Weierstrass (K.). Mathematische Werke. Herausgegeben unter Mitwirkung einer von der K. Preussischen Akademie der Wissenschaften eingesetzten Commission. (In etwa 8 Bänden). Vol. I. Berlin, Mayer & Müller, 1894. 4to. 8 and 356 pp. Mk. 21.00

Zirngast (K.). Die Krümmungslinien des elliptischen Paraboloides als geodätische Ellipsen und Hyperbeln. [Progr.] Weisskirchen, 1894. 8vo. 30 pp.

II. ELEMENTARY MATHEMATICS.

Bruhns (C.). Neues logarithmisch-trigonometrisches Handbuch auf 7 Decimalen. 4te Ausgabe. Leipzig, 1894. 8vo. 24 and 610 pp. Mk. 4.20

Brisse (C.). Cours de géométrie descriptive, à l'usage des élèves de

l'enseignement secondaire moderne. Paris, Gauthier-Villars, 1895. 8vo. 20 and 119 pp. Fr. 7.00

DESSENON (E.). Éléments de géométrie analytique, à l'usage des candidats aux Écoles centrale et navale et des élèves de première année de la classe de mathématiques spéciales. 2e édition. Paris, Hachette, 1894. 8vo. 8 and 523 pp. Fr. 7.50

ELEMENTS DE GÉOMÉTRIE, comprenant des notions sur les courbes usuelles et de nombreux exercices; par les Frères des écoles chrétiennes. 7e édition. Paris, Poussielgue, 1893. 18mo. 11 and 492 pp.

ÉLÉMENTS DE GÉOMETRIE DESCRIPTIVE, avec de nombreux exercices; par F. I. 5e édition. Paris, Poussielgue, 1893. 8vo. 458 pp.

GIRAUD (P.). Algèbre à l'usage des candidats aux baccalauréats de l'enseignement secondaire. Paris, Société d'éditions scientifiques, 1894. 18mo. 2 and 166 pp. Fr. 2.00

GREVY (A.). Compositions données depuis 1872 aux examens de Saint-Cyr: Algèbre et géométrie (énoncés et solutions). 2e édition. Paris, Gauthier-Villars, 1894. 8vo. Illustrated. Fr. 2.50

HIGGS (W. P.). Algebra self-taught. New York, Spon & Chamberlain, 1894. 12mo. 104 pp. Cloth. $0.60

HOCHHEIM (A.). Aufgaben aus der analytischen Geometrie der Ebene. Heft I: Die gerade Linie, der Punkt, der Kreis. A. Aufgaben. B. Auflösungen. 2te, verbesserte Auflage. Leipzig, Teubner, 1894. 8vo. 4 and 192 pp. Mk. 3.20

LAURENT (H.). Traité d'algèbre. Compléments; 4e partie: Théorie des polynomes à plusieurs variables. Paris, Gauthier-Villars, 1894. 8vo. 62 pp. Fr. 1.50

LAURIN (P. G.) Lärobok i Geometri. 2 upplaga. Afdeling I: Plan Geometri. Del I. Lund, 1894. 8vo. 9 and 188 pp. Mk. 3.00

LOPERFIDO (A.). Principi di aritmetica e di geometria, esposti per uso delle scuole elementari e secondarie, conforme agli ultimi programmi governativi. Firenze, 1894. Fr. 2.00

MACDONALD (J. W.). Primary algebra; the teacher's guide. Complete edition. Boston, Allyn & Bacon, 1894. 16mo. 214 pp. Cloth. $0.75

MICHELSEN (P.). Die bestimmten algebraischen Gleichungen des ersten bis vierten Grades. Nebst einem Anhange: Unbestimmte Gleichungen. Für höhere Unterrichtsanstalten, sowie für den Selbstunterricht. Hannover, Meyer, 1894. 8vo. 306 pp. Mk. 4.00

SABIN (S. B.) and LOWRY (C. D.). Elementary lessons in algebra; a text-book for grammar schools. New York, American Book Co., [1894]. 8vo. 128 pp. Cloth. $0.50

III. APPLIED MATHEMATICS.

ANTOMARI (X.). Cours de mécanique, à l'usage des candidats à l'École spéciale militaire de Saint-Cyr. Paris, Nony, 1895. 8vo. 272 pp. Illustrated.

ANTOMARI (X.) and LAISANT (C. A.). Questions de mécanique, à l'usage des élèves de mathématiques spéciales. Paris, Nony, 1895. 8vo. 230 pp. Illustrated.

BAGNOLI (E.) Principi di statica e loro applicazione alla teoria e costruzione degli strumenti metrici. Milano, 1894. 260 pp. Illustrated. Fr. 3.50

CURRY (C. E.). Die Fortpflanzung von Verdichtungsstössen in einem Gase. [Diss.] München, 1894. 8vo. 35 pp.

FORTSCHRITTE DER PHYSIK im Jahre 1888. Dargestellt von der Physikalischen Gesellschaft zu Berlin. Jahrgang 44 (in 3 Abtheilungen). Abtheilung II: Physik des Aethers, redigirt von R. Bornstein. Braunschweig, Vieweg, 1894. 8vo. 45 and 809 pp. Mk. 30.00

FRANKE (A.). Experimentelle Beiträge zur Temperatur-Abhängigkeit der Dielektricitätskonstanten flüssiger Körper. [Diss.] München, 1893. 8vo. 31 pp.

FRESDORF (G.). Die Methoden zur Bestimmung der mittleren Dichte der Erde. [Progr.] Weissenburg, 1894. 4to. 30 pp.

GLAUNER (T.). Ueber den Verlauf von Potentialfunktionen im Raume. [Diss.] Göttingen, 1894. 8vo. 61 pp.

GRASSMAN (H.). Punktrechnung und projektive Geometrie. I: Punktrechnung. (Festschrift.) Halle, 1894. 4to. 28 pp.

HEINKE (C.). Beiträge zur Messung von Inductionscoëfficienten und Capacitäten. [Diss.] München, 1894. 8vo. 67 pp.

KOHLRAUSCH (F.). An introduction to physical measurements. With appendices on absolute electrical measurements. Translated by T. H. Waller and H. R. Procter. 3d edition. London, Churchill, 1894. 8vo. Illustrated. Cloth. 12s. 6d.

LESKY (A.). Die historische Entwicklung des Problems der Saitenschwingung. II. [Progr.] Graz, 1894. 8vo. 31 pp.

NECKER (K.). Zur Ausgleichung von Massenbeobachtungen athmosphärischer Lichterscheinungen. [Diss.] Berlin, 1894. 4to. 31 pp.

PORRO (F.). Elementi di astronomia sferica. Roma. Fr. 5.00

RAYLEIGH (Baron). The theory of sound. 2d edition, revised and enlarged. (In 2 volumes.) Vol. I. London and New York, Macmillan, 1894. 8vo. 490 pp. 12s.

SCHWARZ (W.). Beiträge zur Kenntnis der umkehrbaren Umwandlungen polymorpher Körper. [Diss.] Göttingen, 1894. 4to. 50 pp.

TONDINI DE QUARENGHI. Conférences sur la Réforme du calendrier Julien, l'heure universelle et le méridien initial de Jérusalem. 1e édition. Constantinople, 1894. 8vo. 78 pp. Mk. 3.00

VOLDERAUER (L.). Constructive Behandlung astronomischer Aufgaben. Theil II. Trautenau, 1894. 8vo. 16 pp. Mk. 1.80

WÜLLNER (A.). Lehrbuch der Experimentalphysik. Erster Band: Allgemeine Physik und Akustik. 5te vielfach umgearbeitete und verbesserte Auflage. Leipzig, Teubner, 1895. 8vo. 10 and 1000 pp. Illustrated. Mk. 12.00